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Dr. Kamlesh Kumar
Asst. Prof. (Guest Faculty)
Dept. of mathematics

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Abstract Algebra (Binary operation)

A Binary operation is a calculation that combines two elements (called operands) to produce another element. i.e. Binary operation is an operation of arity two.

A binary operation on a set is an operation whose two domains and the codomain are the same set. Examples include the familiar arithmetic operations of addition, subtraction, multiplication. Other examples are readily found in different areas of mathematics, such as vector addition, matrix multiplication and conjugation in groups.

An operation of arity two that involves several sets is sometimes also called a binary operation. For example, scaler multiplication of vector spaces, takes a scalar and a vector to produce a vector, and scalar product product takes two vectors to produce a scalar. Such binary operations may be called simply binary functions.

Binary operations are the keystone of most algebraic structures, that are studied in algebra, in particular in semigroups, monoids, groups, rings, fields, and vector spaces.

A binary operation on a set S is a mapping of the elements of the Cartesian product $S \times S$;

$f: S \times S \to S$.

Because the result of performing the operation on a pair of elements of S is again an element of S, the operation is called a closed binary operation ons.

If f is not a funtion, but a partial function, then f is called a partial binary operation. For ~~that~~ Example, division of real numbers is a partial binary operation, because one cannot divide by zero; a/0 is undefined for every real number a. In both universal algebra and model theory, binary operations are required to be defined on all of $S \times S$, Sometimes, especially in computer science, the term binary operation is used for any binary function.

Properties:- Typical examples of binary operation are the addition (+) and multiplication (×) of numbers and matrices as well as composition of functions on a single set. For instance,

(i) On the set of real numbers R, $f(a,b) = a+b$ is a binary operation since the sum of two real numbers is a real number

(ii) On the set of natural numbers N, $f(a,b) = a+b$ is a binary operation since the sum of two natural numbers is a natural number.

(iii) On the set $M(2,R)$ of 2×2 matrices with real entries, $f(A,B) = A+B$ is a binary operation since the sum of two such matrices is a 2×2 matrix.

(iv) On the set $M(2,R)$ of 2×2 matrices with rea[l] entries, $f(A,B) = AB$ is a binary operation since th[e] product of two such matrices is a 2×2 matrix.

(v) For a given set C, let S be set of all functions $h: C \to C$. Define $f: S \times S \to S$ by $f(h_1, h_2)(c) = (h_1 \circ h_2)(c) = h_1(h_2(c))$ for all $c \in C$, the composition of two functions h_1 and h_2 in S. Then f is a binary operation since the compo[-]sition of the two functions is again a functio[n] on the set C (that is, a member of S).